A TLM Algorithm for Wave Propagation In Human Fat and Muscle Tissues With a Cole-Cole Dispersion Model

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Abstract

A transmission line method (TLM)algorithm is proposed for electromagnetic (EM) wave propagation in biological tissues with the Cole-Cole dispersion Model. A linear behavior of the polarization current during the time step is assumed. The polarization current density is approached using Lagrange extrapolation polynomial, and the fractional derivation is obtained according to Riemann definition. Reflection coefficients at an air/muscle interfaces simulated in a 1-D domain are found in good agreement with those obtained from the analytic model.

1. Introduction

In the last two decades there has been a renewed interest in the interaction between biological tissues and electromagnetic field at microwave frequencies due to new promising applications of this technology in biomedical engineering. the Cole-Cole modelproposed in [1] as:

$$\epsilon_r(\omega) = \epsilon_\infty + \sum_{p=1}^{p} \frac{\Delta\epsilon_p}{1 + (j\omega\tau_p)^{\alpha_p}} \tag{1}$$

where ϵ_∞ is the optical relative permittivity, $\Delta\epsilon_p = \epsilon_s - \epsilon_\infty$ is the amplitude of the $p-th$ pole, ϵ_s is the static relative permittivity, ω is ths angular frequency , τ_p is the relaxation time, α_p the parameter that indicates the broadening of the dispersion for this pole, which in the Cole-Cole model must satisfy $0 < \alpha_p < 1$, The consequence is a fractional order differentiation in the time domain[2].

2. FORMULATION AND EQUATIONS

In the Cole-Cole model the relationship between the polarization current related to the p_{th} pole and the electric field in time domain is given by:

$$\mathbf{J}_p(t) + \tau^{\alpha_p} D^{\alpha_p} \mathbf{J}_p(t) = \epsilon_0 \Delta\epsilon_p \frac{\partial \mathbf{E}(t)}{\partial t} \tag{2}$$

In order to obtain the descritized expression of $\mathbf{J}_p(t)$, the polarization current vector can be approached using a Lagrange interpolation in the time interval $t_i < t < t_j$ .Then performing a Reimann[3] fractional derivative the update equation for the polarization current is derived:

$$\mathbf{J}_p^{n+1} = -\frac{\phi_p}{\psi_p}\mathbf{J}_p^n + \frac{\zeta_p}{\psi_p}(\mathbf{E}^{n+1} - \mathbf{E}^n) \tag{3}$$

where the update parameters ψ_p, ϕ_p and ζ_p are:

$$\psi_p = \frac{1}{2} + \frac{\tau_p^{\alpha_p}}{\Delta t}\frac{\Gamma(2)}{\Gamma(2-\alpha_p)} t^{1-\alpha_p} - n\frac{\tau_p^{\alpha_p}}{\Gamma(1-\alpha_p)} t^{-\alpha_p} \tag{4}$$

$$\phi_p = \frac{1}{2} - \frac{\tau_p^{\alpha_p}}{\Delta t}\frac{\Gamma(2)}{\Gamma(2-\alpha_p)} t^{1-\alpha_p} + \frac{(n+1)\tau_p^{\alpha_p}}{\Gamma(1-\alpha_p)} t^{-\alpha_p} \tag{5}$$

$$\zeta_p = \frac{\epsilon_0 \Delta\epsilon_p}{\Delta t} \tag{6}$$

To get the update equation of the electric field components we start from the Maxwell-Ampere equation, and by including the conductivity term :

$$\nabla \times \mathbf{H} = \epsilon_0\epsilon_\infty \frac{\partial \mathbf{E}}{\partial t} + \sigma_0 \mathbf{E} + \sum_{p=1}^{p} \mathbf{J}_p \tag{7}$$

where σ_0 is the static ionic conductivity, the updated electric field is given by:

$$\begin{aligned} E_{x,y,z}^{n+1} &= E_{x,y,z}^n (1 - \frac{2\sigma_0\Delta t}{D}) \\ &- \sum_p \frac{(1 - \frac{\phi_p}{\psi_p})\Delta t}{D} J_{px,y,z}^n + \frac{2\Delta t}{D}(\nabla \times \mathbf{H})_{x,y,z}^{n+\frac{1}{2}} \end{aligned} \tag{8}$$

$$D = 2\epsilon_0\epsilon_\infty + \sum_p \frac{\zeta_p \Delta t}{\psi_p} + \sigma_0 \Delta t \tag{9}$$

In the TLM method the simulation domain is discretized in cells where a series of uniform transmission lines model the propagation medium and the unit cell circuit referred to as symmetrical condensed node(SCN).

The voltage at the center of the SCN node, as deduced from the conservation laws[4], becomes :

$$\begin{aligned} V_{x,y,z}^{n+1} &= V_{x,y,z}^n + \frac{1}{4 + Y_{ocx,y,z}}({}_sV_{x,y,z}^{n+1} + {}_sV_{x,y,z}^n) \\ &+ \frac{4}{4 + Y_{ocx,y,z}} \frac{\Delta l \Delta t}{\epsilon_0} (\nabla \times \mathbf{H})_{x,y,z}^{n+\frac{1}{2}} \end{aligned} \tag{10}$$

where $Y_{ocx,y,z}$ indicates the normalized admittance of the open circuit stub added to the node.

The update equation for voltage sources at the center of the node:

$${}_sV_{x,y,z}^{n+1} = -{}_sV_{x,y,z}^n + C_1 V_{x,y,z}^n + \sum_p C_{2p} J_{px,y,z} \tag{11}$$

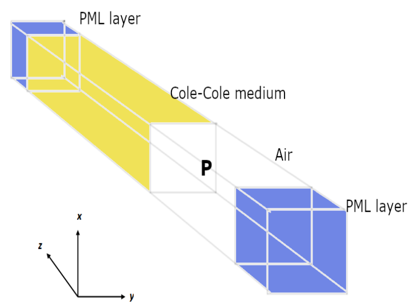

Figure 1: 3D view of the computation domain , incident and reflected fields are calculated at point P.

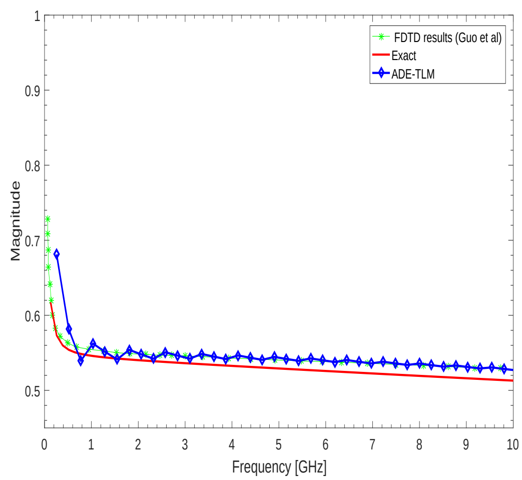

Figure 2: Reflection coefficient (dB) at normal incidence on an air/Fat interface obtained by the proposed algorithm compared to the FDTD results [2] and the exact solution

with update constants:

$$C_1 = -\frac{4\sigma_0 \Delta t}{\epsilon_0} \qquad C_{2p} = -\frac{2(1-\frac{\phi_p}{\psi_p})\Delta t \Delta l}{\epsilon_0} \tag{12}$$

3. SIMULATION AND RESULTS

The simulation domain as depicted in Figure 1 consists of a grid of 1000 cells interconnected on the z axis truncated by an unsplit PML layer [5] of 10 cells at the beginning and the end.500 cells were used to model the Cole-Cole medium and the remaining cells were used to model the air is. The cell dimension Δl satisfies the stability condition $\Delta t = \Delta l/2c$, the time step is set as $\Delta t = 0.125ps$. The fourth order Cole-Cole parameters for Cole-Cole media are taken as in [2].

In Figures 2 and 3 the simulation results are compared to the theoretical reflection coefficient and to the FDTD results at the air/Cole-Cole medium interface A good agreement over the whole frequency band is observed.

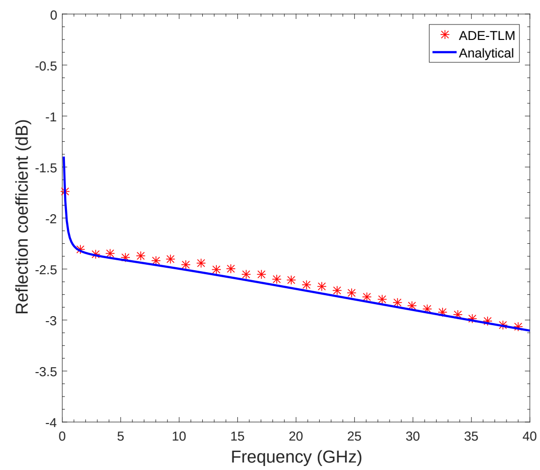

Figure 3: Reflection coefficient (dB) at normal incidence on an air/Muscle interface

4. CONCLUSION

In this paper, a TLM algorithm is presented to study the electromagnetic wave propagation in biological tissues using Cole-Cole dispersion model. A polynomial extrapolation is used to approach the polarization current and a Reimann definition iss used to perform the fractional derivation . This approximation reduces considerably both the numerical cost of the simulation since only two previous values of the J field are needed at each iteration. The presented results indicate the accuracy of our approach.

References

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