Accurate and Efficient Analysis of the Electromagnetic Scattering from a thin disk

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Abstract

An effective semi-analytical technique for the analysis of the scattering from a thin disk is briefly presented. The problem, formulated as sets of one-dimensional integral equations in the vector Hankel transform domain, is discretized by means of Helmholtz decomposition and Galerkin method with complete sets of orthogonal eigenfunctions of the most singular part of the integral operators, reconstructing the physical behavior of the fields, as expansion bases. In this way, fast-converging Fredholm second-kind matrix equations are obtained.

1. Introduction

The analysis of the electromagnetic scattering from finite objects is frequently carried out by means of integral equation formulations because the unknowns are defined on finite supports and the radiation condition is taken into account by a suitable choice of the Green's function of the problem. For such equations approximate solutions are generally obtained by resorting to discretization techniques. However, the existence of the solution of an arbitrary integral equation cannot be generally stated. Moreover, if the solution exists, the convergence of a discretization scheme cannot be generally established. Indeed, Fredholm's theory can be applied only if the operator is the superposition of a continuously invertible operator and a completely continuous operator [1].

When dealing with a first-kind integral equation or a hyper-singular second-kind integral equation, a Fredholm second-kind equation can be obtained by analytically inverting the most singular part of the integral operator [2], [3]. On the other hand, Fredholm theory can be applied to the matrix equation obtained by means of the Galerkin method with a complete set of orthogonal eigenfunctions of a suitable operator, containing the most singular part of the original integral operator, as expansion basis [2], [3]. Under such conditions, Galerkin-projection acts as a perfect analytical preconditioner for the considered integral equation. For this reason, this approach, called method of analytical preconditioning, has been widely used for the analysis of propagation, radiation, and scattering problems [4-12].

The aim of this paper is the analysis of the scattering from a thin disk in a homogeneous medium by means of a method recently developed by the author and belonging to the class of the methods of analytical preconditioning [13-17]. Surface integral equations for the effective electric and magnetic currents are obtained by imposing the generalized boundary conditions on the disk surface. Due to the revolution symmetry of the problem, all the involved functions are conveniently expanded in Fourier series and the surface integral equations are recast as sets of one-dimensional integral equations in the spectral domain for the harmonics of the vector Hankel transform (VHT) of the effective currents. The surface curl-free and divergence-free contributions of the effective currents are assumed as new unknowns in order to handle scalar unknowns in the VHT domain. Galerkin method with complete sets of orthogonal eigenfunctions of the most singular parts of the integral operators reconstructing the behavior of the fields at the edge and around the center of the disk as expansion bases are used to discretize the integral equations, thus leading to fast converging Fredholm second-kind matrix equations.

2. Formulation and Solution of the Problem

A disk of radius a and thickness τ immersed in a homogeneous medium is such that $\tau \ll a$ and $\tau \ll \lambda$, where λ is the wavenumber in the external medium. A cylindrical coordinate system (ρ,ϕ,z) with the origin at the center of the disk and the z axis orthogonal to it is introduced. An incident field $\left(\underline{E}^{inc}(\underline{r}),\underline{H}^{inc}(\underline{r})\right)$, where $\underline{r} = x\hat{x} + y\hat{y} + z\hat{z}$, impinges onto the disk surface generating a scattered field $\left(\underline{E}^{sc}(\underline{r}),\underline{H}^{sc}(\underline{r})\right)$ such that the tangential component of the total field satisfies the generalized boundary conditions on the disk surface, i.e., [18]

$$\hat{z}\times\left(\underline{E}^{inc}(\rho,\phi)+\underline{E}^{sc}(\rho,\phi)\right)\times\hat{z} = R\ \underline{J}(\rho,\phi), \quad (1)$$

$$\hat{z}\times\left(\underline{H}^{inc}(\rho,\phi)+\underline{H}^{sc}(\rho,\phi)\right)\times\hat{z} = S\ \underline{M}(\rho,\phi), \quad (2)$$

for $\rho \le a$, where

$$\underline{J}(\rho,\phi)=\hat{z}\times\left(\underline{H}^{sc}(\rho,\phi,0^+)-\underline{H}^{sc}(\rho,\phi,0^-)\right), \quad (3)$$

$$\underline{M}(\rho,\phi)=-\hat{z}\times\left(\underline{E}^{sc}(\rho,\phi,0^+)-\underline{E}^{sc}(\rho,\phi,0^-)\right), \quad (4)$$

are the effective electric and magnetic currents, respectively, and R and S are the electric and magnetic resistivities of the disk, respectively. The uniqueness of the solution is guaranteed by requiring that the total field satisfies the edge conditions and the radiation condition.

The revolution symmetry of the problem allows to expand all the involved functions in series of orthogonal azimuthal harmonics. Hence, the surface integral equations for the effective currents can be equivalently reduced to infinite sets of independent one-dimensional integral equations in the spectral domain having as unknowns the VHT of the harmonics of the effective currents [13].

According to the Helmholtz decomposition, each current is written as the superposition of a surface curl-free contribution and a surface divergence-free contribution, which are assumed as new unknowns in order to handle scalar unknowns in the VHT domain [15].

The obtained integral equations are discretized by means of the Galerkin method. The behavior of the *n*-th harmonic of the effective currents at the edge and around the center of the disk is correctly reconstructed by expanding the unknowns in the following complete series of Bessel functions [16, 19], i.e.,

$$\tilde{f}^{(n)}(w) = \sum_{h=-1+\delta_{n,0}}^{+\infty} \gamma_h^{(n)} \sqrt{2p_h}\, \frac{J_{p_h}(aw)}{w^{\eta}}, \qquad (5)$$

where $\delta_{n,m}$ is the Kronecker delta, $\gamma_h^{(n)}$ denote the expansion coefficients, $p_h = |n| + 2h + \eta + 1$ and η depends on the edge behavior. With such a choice, the integral equations are reduced to fast converging Fredholm second-kind matrix equations. Moreover, the coefficient matrix elements, which are one-dimensional improper integrals of oscillating functions, are efficiently evaluated by means of the analytical procedure in the complex plane developed in [14, 15].

3. Numerical Results

An approximate solution of the problem can be obtained by truncating the infinite matrix equations. To show the fast convergence of the presented method, the following normalized truncation error is introduced

$$\mathrm{err}_N(M) = \sqrt{\sum_{n=-N+1}^{N-1} \left\| \mathbf{x}_{M+1}^{(n)} - \mathbf{x}_M^{(n)} \right\|^2 \Big/ \sum_{n=-N+1}^{N-1} \left\| \mathbf{x}_M^{(n)} \right\|^2}, \qquad (6)$$

where $2N-1$ is the number of considered harmonics estimated as in [20], $\|\cdot\|$ is the usual Euclidean norm and $\mathbf{x}_M^{(n)}$ is the vector of the expansion coefficients evaluated by using M expansion functions for each unknown.

In Figure 1a, $\mathrm{err}_N(M)$ is plotted for $N = 15$ in the case of a TM polarized plane wave, with incident angles $\theta_i = 30°$ with the z axis and $\phi_i = 0°$ with the x axis in the xy plane, impinging onto a resistive disk with $a = 2\lambda$ and $R = 100\Omega$. In this case, the generalized boundary conditions reduce to equation (1). As clearly shown, the convergence is very fast. Indeed, few expansion functions are needed to accurately reconstruct the solution. This behavior can be further appreciate in Figures 1b and 1c, where the effective electric current and the bistatic radar cross-section (BRCS) in the plane $\phi = 0°, 180°$, obtained for $M = 9$ so that the error is less than 10^{-3}, are plotted and compared with very good agreement with the results provided by CST Microwave Studio (CST-MWS). It is worth noting that, in reconstructing the solution in the case examined, the proposed method is 30 times faster than CST-MWS.

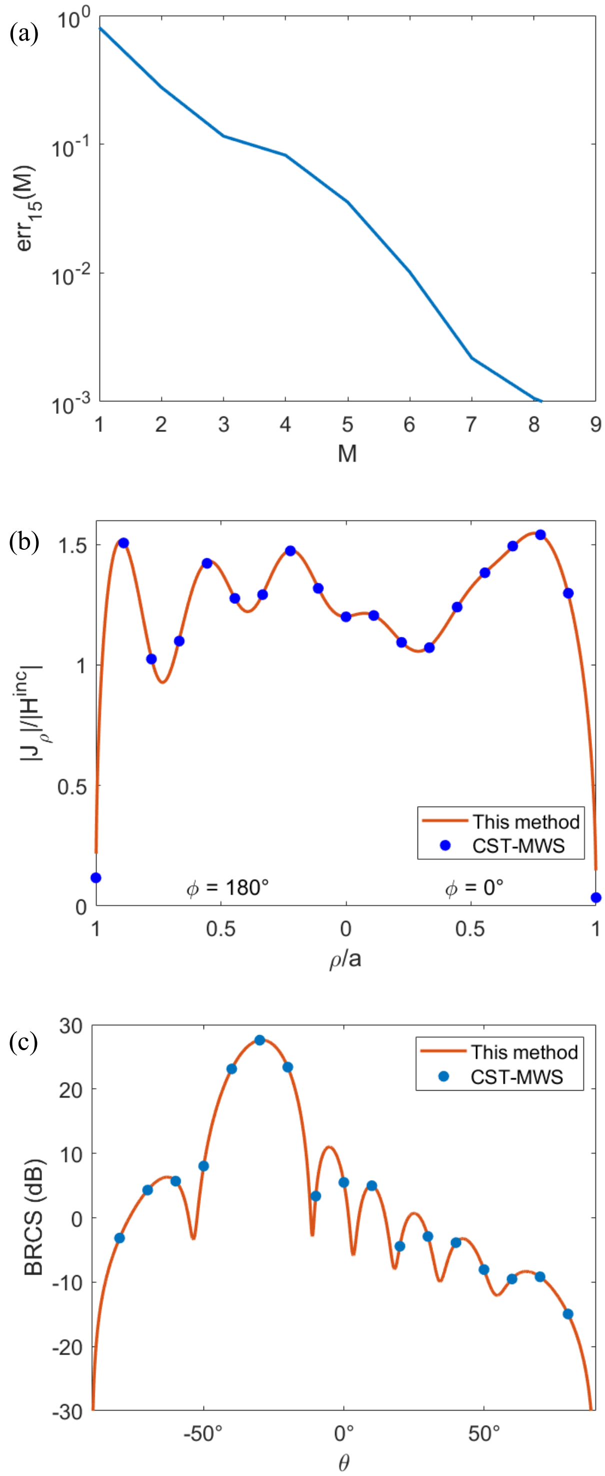
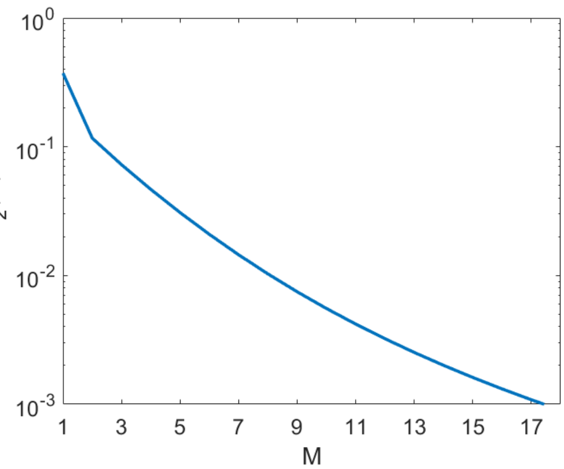


Figure 1. (a) Normalized truncation error, (b) effective electric current, and (c) bistatic radar cross section of a resistive disk with

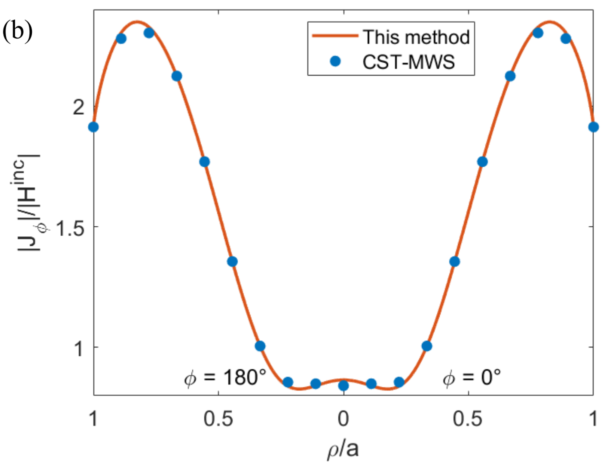
$a = 2\lambda$ and $R = 100\Omega$ when a plane wave impinges onto the disk surface with $\theta_i = 30°$, $\phi_i = 0°$ and TM polarization.

In Figure 2a, $\mathrm{err}_N(M)$ for a dielectric disk of radius $a = 3\lambda/(2\pi)$, thickness $\tau = 0.05a$ and relative dielectric permittivity $\varepsilon_r = 10.5 - j0.3$, is plotted as a function of M for normal to the disk incidence, i.e., $\theta_i = 0°$, with $\underline{E}_0 = E_0\hat{y}$. In this case, only the harmonics for $n = \pm 1$ contribute to the field's representation, i.e., $N = 2$, and the resistivities can be approximated by means of the formulas devised for high-index contrast materials shown in [18]. The convergence is, again, very fast, and less than 1 min is needed to accurately reconstruct the solution. For the sake of completeness, in Figures 2b, 2c and 2d the effective electric current, the effective magnetic current and the BRCS, obtained for $M = 18$ so that the error is less than 10^{-3}, are plotted in the plane $\phi = 0°, 180°$ and compared with the results provided by CST-MWS. The agreement is quite good. It is interesting to note, however, that CST-MWS requires a computation time of 80 mins to reconstruct the plotted solution. After all, to the best of the authors knowledge, CST-MWS does not provide a 2D model for thin dielectric objects and an accurate 3D simulation has turned out to be time-consuming and particularly burdensome in terms of memory requirement.

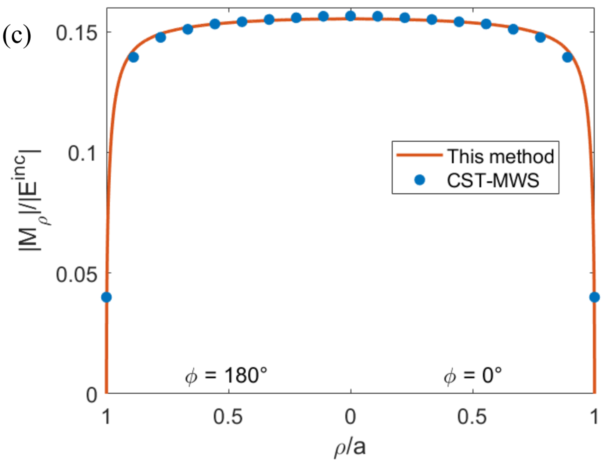
(a)



(b)



(c)



(d)

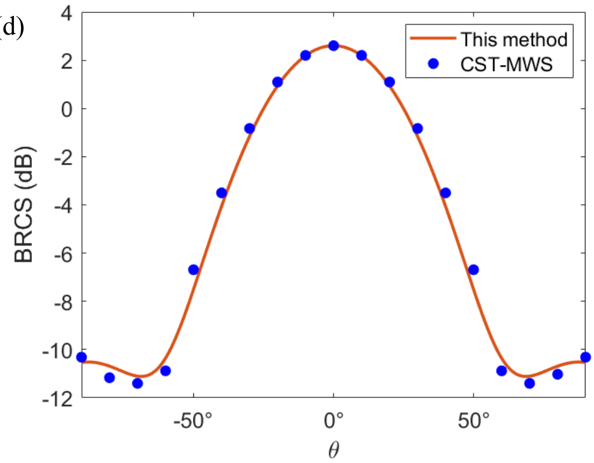


Figure 2. (a) Normalized truncation error, (b) effective electric current, (c) effective magnetic current, and (d) bistatic radar cross section of a dielectric disk with $a = 3\lambda/(2\pi)$, $\tau = 0.05a$ and $\varepsilon_r = 10.5 - j0.3$, when a plane wave normally impinges onto the disk surface with $\underline{E}_0 = E_0\hat{y}$.

4. Conclusions

The analysis of the electromagnetic scattering from a thin disk has been successfully carried out by means of a regularizing and fast converging method based on the Helmholtz decomposition and the Galerkin method. Comforting preliminary results show that the proposed method can be generalized to study more complex structures such as arrays of disks in non-homogeneous media.

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