A direct and an inverse domain decomposition method applied to Fourier modal method: simulation of large scale device response

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Abstract

A parallel spectral modal method is introduced for the frequency-domain Maxwell's equations. The method is applied to compute electromagnetic field through a large-scale surface, using the Aperiodic Fourier Modal Method (AFMM). In the proposed domain decomposition methods, a large-size surface is squared onto sub-cells. And a projector, the link between the large problem and the sequence of the small one, is defined, from the sequence of eigenfunctions of both large and small problems. This projector allows associating univocally the spectrum of any electromagnetic field component of an electromagnetic problem stated on a large-size domain with its footprint on the small-scale problem eigenfunctions, in the spectral domain. And the spectrum of the electromagnetic field component on each sub-cell may be simulated, locally with a coast that is not dependent on the number of the sub-cells, making it suitable for parallel computing. 2D validation cases are demonstrated by first, computing the electromagnetic field radiated by an electric dipole. Second, a 2D dielectric monochromatic metalens topology optimization is successfully proposed.

1. Introduction

In most electromagnetic problems, the number of unknowns required to simulate the electromagnetic components grows with respect to the size of the device geometry compared to the operating wavelength. In this context, for any rigorous full-wave methods, tackling such problems beyond limits fixed by computation time and memory, may become an infeasible task. To ensure fast and reliable numerical solutions, special care must be focused on, firstly the way the code is implemented, and, secondly, on the numerical method used to solve equations at hand.

To address the first point raised above, for a given electromagnetism problem stated on a large-scale domain, one of the intuitive methods consists in sub-structuring this computational domain. This technique consists in squaring the whole domain into a sequence of sub-domains, leading to a construction of sub-domains and interfaces matrices. The solution on each sub-domains can be performed using a parallel strategy or an iterative one. In the iterative process, the solution of the equation is only performed after solving interfaces or boundaries equations. This approach can be fruitfully considered within the context of iterative domain decomposition techniques such as the Schwarz method. However, this technique may require excessive memory and time consuming, when the number of interfaces increases. In the second strategy, suitable for parallel programming, one stitches the grid of the subdomains without applying any boundary condition. To minimize the stitching error, in this case, the size of each sub-domain may be kept relatively large compared to the wavelength, or they can also be geometrically separated each from other by a few gaps empirically determined.

Now let us focus on the second point raised earlier *i.e.* the numerical method itself. In photonics, a large class of electromagnetic field simulation is often required to solve partial differential equations (PDE) obtained from Maxwell's equations and a set of *ad-hoc* boundary conditions. *A priori*, there is no unique and universal method that allows to efficiently solve PDEs obtained from Maxwell's equations in the general case. Among all these methods, certain clearly appears to be the most commonly used, either by their simplicity of implementation, or by their ability to take into account complex geometry problems, and finally either because they are the most efficient in terms of numerical accuracy. These are: finite difference methods, finite element methods, or spectral methods [1]. These methods can be categorized into a direct-space method and spectral method. All these numerical methods involve different amounts of theoretical effort yielding different computational efficiencies. It is easy to understand that improving the efficiency of the numerical implementation and process will be often paid for with more pre-computational analytical effort and electromagnetic theory use. Spectral methods aim to approximate an unknown function, solution of differential or integral equation, by a finite sum of so-called basis functions. This method appears as an alternative to a direct-space method as finite-difference and finite element methods for numerical solving of partial differential equations. Studying diffraction by a one-dimensional dielectric and metallic lamellar grating, Lalanne in [2] compares the Fourier modal method equipped with the adaptative spatial resolution to the finite difference method. It appeared clearly in their study that the Fourier spectral modal method performs better than the frequency domain finite difference method. In spite of previous developments, and contrary to spatial-method as finite difference method stated in time [3, 4] or frequency domain, speedup of research of numerical solutions to large scale problems thanks to a spectral method in electromagnetism poses some major challenges. And implementation of the sub-structuring technique seems to be a way to overcome this drawback. In [5] authors introduce a strategy for optimizing large area metasurface using aperiodic Fourier modal method and by stitching together individually optimized sections of the metasurface. The proposed approach is successfully applied to design a 1D large-scale dielectric metalens using the topology optimization method. Each element of the metalens is designed as a deflector with a specific additional phase condition.

In this paper, we propose a mathematical formalism based on a projector enabling us to project any eigenvectors defined on a domain Ω into a sequence of eigenfunctions of smaller boundary value problems defined on subdomains Ω_{ij}. In the proposed domain decomposition methods, a large-scale surface is, obviously, squared onto sub-cells. Hence, a projector, a link between a large problem and a small one, is introduced. This projector, defined from the sequence of eigenfunctions of both large and small problems enables to associate univocally the spectrum of any electromagnetic field component related to the large-size problem, with its restriction on the small-size problem. The spectrum of the electromagnetic field component on each sub-cell may then be simulated independently, making the proposed approach to be, optimally and simply integrated into a parallel computing scheme, for the research of solutions of larger size systems. First, we successfully demonstrate the ability of the proposed method to describe accurately a field radiated by an electric dipole in free space. We show that the solver can perform simulations of the field on each sub-cell then on a large area of metalenses in a record time. Second, we make use of this capability to yield high-performance improvement over a high-refractive-index-deeply-etched Fresnel plate zone design through optimization.

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