Critical distances for near-ground propagation: application to dipole antennas

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Abstract

This paper deals with the problem of near-ground wave propagation, in particular with the assessment of the region in which the near-ground wave becomes the dominant component. The critical distances are estimated as a function of the link parameters in the case of half-wave dipole antennas.

1. Introduction

In the context of smart cities, wireless sensor networks (WSN) have been widely deployed for environment purposes or surveillance. In some cases, the sensors are placed close to a conductive interface in order to collect data of interest. The radiation of a near-ground sensor invokes the famous Sommerfeld half-space problem. The Sommerfeld formulation is widely accepted in the community and can be validated by rigorous mathematical treatment [1–5]. The steepest descent technique provides an infinitesimal model describing the electric field radiated by an elementary vertical electric dipole (VED) over a lossy interface [6]. Recently, in [6] the existence of a region of interest for the surface component has been highlighted. According to [1,2,6], the E_z component of the electric field follows a two-phase decay: a region where the attenuation is about 10 dB/decade and a second where it is about 20 dB/decade. Analytical expressions of the two critical distances have been proposed for a VED case [1, 2, 6]. In this paper, we propose an extension for the propagation model of the infinitesimal vertical dipole near the ground. We also introduce an empirical method to determine the critical distances in the case of a half-wave dipole antenna. Finally, the conditions of emergence of the surface component are determined, in order to verify the possibility for the two-ray method [7] to predict the field in certain cases with a good precision.

2. Infinitesimal vertical dipole

The Sommerfeld half-space problem [1] for dipole antennas involves image theory. As we can see in fig. 1, the model of the problem is composed of an infinite lossy surface dividing the space into two different media, the emitting and the receiving antennas are placed in medium 1. The interface is further replaced by the image of the emitting antenna with respect to the interface.

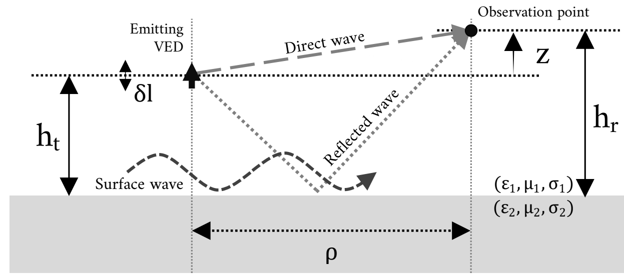

Figure 1: VED radiating over a lossy half-space

Here δl is the infinitesimal length of the emitting VED, h_t the height of the emitting VED, h_r the height of the observation point, ρ the radial distance between the emitting VED and observation point, z the vertical distance between the emitting VED and the observation point ($z \in]-h_t, \infty[$), and finally ε_i, μ_i and σ_i are the electromagnetic characteristics of the propagation medium ($i = 1$) and the lossy medium ($i = 2$). Fig. 1 suggests that besides the classic direct and reflected waves, one needs to include a third component in order to model the wave propagation properly. This term which is due to the proximity to the interface may commonly be called the "surface wave" which attenuates rapidly above the interface.

As described in [6], the z component of the electric field radiated by an infinitesimal vertical dipole (E_z^{VED}) is given by the asymptotic approximation below:

$$E_z^{\text{VED}} = -jI_0\delta l\eta_1 k_1 \Bigg(\sin^2(\theta)\frac{e^{-jk_1 r}}{4\pi r} + \Gamma\sin^2(\theta_I)\frac{e^{-jk_1 r_I}}{4\pi r_I} \underbrace{-jF\sqrt{2\sin^7(\omega_p)}\frac{n^2}{1-n^2}\cos(\omega_p)\frac{e^{-jk_1 r_I}}{4\pi r_I}}_{\text{Surface component}}\Bigg) \quad (1)$$

where the first two terms can be provided by the classic two-ray model as below :

$$E_{z\ \text{two-ray}}^{\text{VED}} = -jI_0\delta l\eta_1 k_1 \left(\sin^2(\theta)\frac{e^{-jk_1 r}}{4\pi r} + \Gamma\sin^2(\theta_I)\frac{e^{-jk_1 r_I}}{4\pi r_I}\right) \quad (2)$$

In (1) and (2), the angles and radiuses (r, θ) and (r_I, θ_I) are functions of ρ and z and are related to the relative positions of the observation point, the VED and the VED image (see fig. 1). The other parameters are summarized in table 1.

Table 1: Parameters in equation (1)

Parameter	Description
Γ	Reflection coefficient of the interface
F	Sommerfeld attenuation term [6]
$I_0\delta l$	Dipolar moment of the VED
η_1	Medium impedance
k_1	Wavenumber
$\omega_p = \arccos(\sqrt{\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}})$	Pole of the reflection coefficient
$n = \sqrt{\varepsilon_2/\varepsilon_1}$	Refractive index (provided that the first medium is vacuum)
$\tau = jk_1r_I[\cos(\omega_p - \theta_I) - 1]$	Numerical distance [6]

3. Half-wave vertical dipole

3.1. Current distribution

The accurate current distribution on a finite-length vertical dipole (VD) above an interface should be obtained by a rigorous numerical method such as the Method of Moments (MoM) [8]. In the case of a half-wave thin dipole in free space, a sinusoidal approximation can describe the current distribution with a good precision [9]. If the half-wave dipole is fed by a 1 A current, the current distribution along the dipole would be as follows:

$$I(z') = \sin\Big(k_1\big(\frac{\lambda}{4} - |z'|\big)\Big) \quad (3)$$

where $z' \in [-\lambda/4, \lambda/4]$ is the vertical position over the dipole.

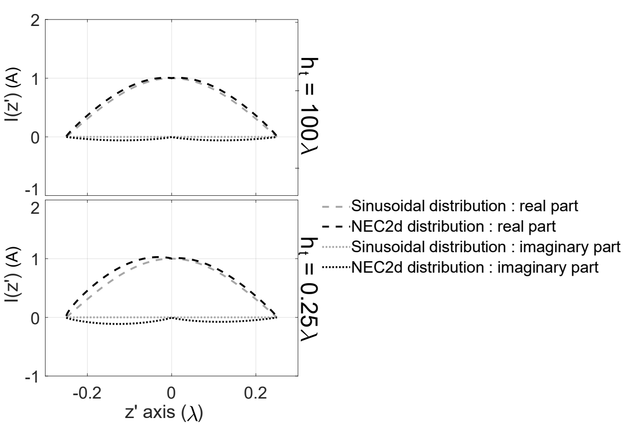

Figure 2: Current distribution on a half-wave vertical dipole above a PEC

In order to check the validity of the sinusoidal approximation to predict the current values over a half-wave dipole above an interface, the current distribution computed by NEC2d [10] is compared with the sinusoidal one. We consider a working frequency of 100 MHz and a Perfect Electric Conductor as interface. The results are given by Fig. 2 and the Relative Root Square Errors (RRSE) (described by eq. 4) are summarized in Table 2 for two different heights.

$$\mathrm{RRSE} = \frac{\|I_{\mathrm{NEC}}(z') - I(z')\|_2}{\|I_{\mathrm{NEC}}(z')\|_2} = \sqrt{\frac{\int_{-\lambda/4}^{+\lambda/4} |I_{\mathrm{NEC}}(z') - I(z')|^2\, dz'}{\int_{-\lambda/4}^{+\lambda/4} |I_{\mathrm{NEC}}(z')|^2\, dz'}} \quad (4)$$

Table 2: RRSE on the current distribution of the half-wave VD at two different heights

Configuration	RRSE
$h_t = 100\lambda$	**8.713%**
$h_t = 0.25\lambda$	**14.42%**

Fig. 2 and Table 2 show that there is a slight difference between NEC2d current distribution and the sinusoidal one. This difference is mainly due to the fact that the sinusoidal distribution is omitting the imaginary part of the current. In addition to this, we notice that the RRSE increases as the dipole gets closer to the interface while the current tends to get asymmetric.

3.2. Extension of the infinitesimal model

Fig. 3 shows that the VD of length $L = \lambda/2$ is decomposed into a infinite set of VEDs of length δl, the current of each weighted according to the sinusoidal distribution given in (3). Their radiated electric fields using (1) are summed up at the reception point based on the superposition principle.

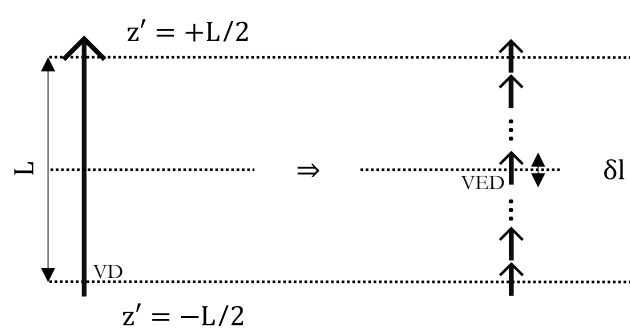

Figure 3: Approximation of a half-wave VD by a set of VEDs

The computation of the total electric field radiated by a half-wave VD, following the superposition principle, leads us to an integral. By rewriting (1) in the following compact form $E_z^{\mathrm{VED}} = I_0\delta l f(\rho, z)$, the expression of E_z^{VD} is consequently given by:

$$E_z^{\mathrm{VD}} = \int_{-\lambda/4}^{+\lambda/4} I(z') f(\rho, z - z') dz' \quad (5)$$

The extended model would be comparable to the infinitesimal model in terms of the electric field levels (especially in far-field) provided that the dipolar moments of both dipoles are equivalent. This equivalence is established by the following equation:

$$I_0 \delta l = \int_{-\lambda/4}^{+\lambda/4} I(z') dz' \tag{6}$$

By applying (3) to (6), we finally obtain:

$$I_0 \delta l = \frac{\lambda}{\pi} \tag{7}$$

In the following sections, this equivalence is automatically used to estimate the dipolar moment of the VED.

3.3. Comparison between the infinitesimal and the extended models

In order to highlight the differences between the infinitesimal and extended models, both models are tested and compared. We choose a working frequency of $f = 868$ MHz, at the emitting and receiving heights of $h_t = h_r = 0.5\lambda$, the characteristics of the lossy medium being $\varepsilon_r = 1$ and $\sigma_2 = 100$ S/m. The results are shown in Fig. 4.

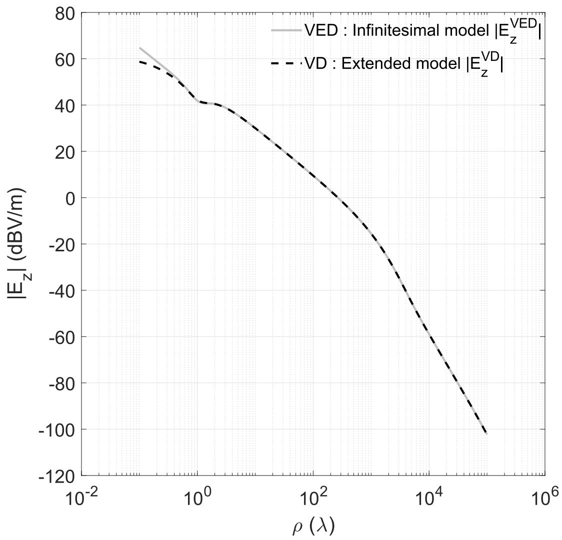

Figure 4: Evolution of $|E_z|$ of both models according to ρ

This result shows that as long as the equivalence seen in (6) is applied, both models overlap over a large range of ρ distances, especially in far-field. As a first conclusion, we can say that a half-wave VD shows the same radiation behavior in far-field as a VED, provided that the dipolar moments are equivalent. This result will be subject to further investigations in future works.

4. Critical distances

4.1. Definition

In order to identify the effect of the surface component on the E_z field, we introduce the critical distances and the region of interest. The determination of theses distances is based on an empirical approach, relying on the two-ray propagation model, and allowing us to mark out a region of interest, where the surface component becomes predominant.

We define the region of interest as to be the region delimited by the critical distances, i.e. the region starting from the distance where $|E_{z\text{ surface}}|$ becomes greater than $|E_{z\text{ two-ray}}|$ to the distance where $|E_{z\text{ surface}}|$ attains $|E_{z\text{ surface}}|_{\max}$. We also define the quantity $\Delta\rho$, representing the extent of the region of interest, and ΔE_z being the maximum gap between $|E_z|$ and $|E_{z\text{ two-ray}}|$.

4.2. Parametric study

In this section, we expose a parametric study using the extended model in the case of a half-wave dipole. The output parameters, i.e. $\Delta\rho$ and ΔE_z, are computed according to the simultaneous variation of the problem's parameters. The main goal is to bring out the conditions of emergence of the surface component. Having these conditions in hand, one knows in advance in which case the use of the two-ray model could guarantee a good precision and thus be sufficient. The parametric study is carried out by considering two configurations for a half-wave VD ($L = \lambda/2$) shown in Table 3.

Table 3: Parameters of the test configurations

	f (MHz)	h_t (λ)	z (λ)	ε_r	σ_2 (S/m)
Config. 1	100	$[0.25, 1.25]$	0	1	$[5, 500]$
Config. 2	$[1, 100]$	$[0.25, 1.25]$	0	1	5

The results, in the form of maps, are given by Fig. 5.

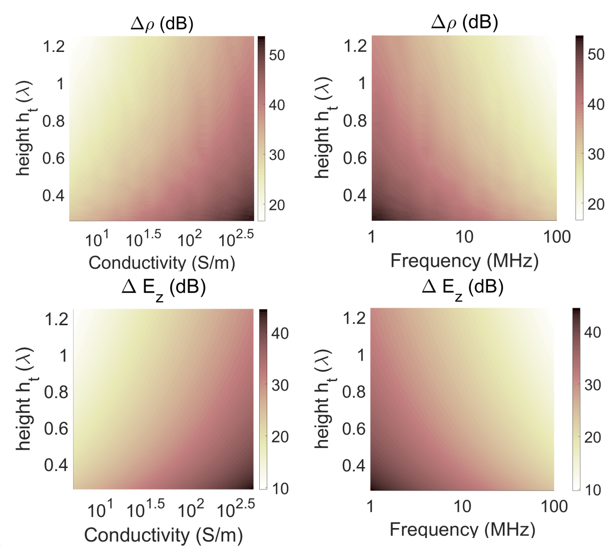

Figure 5: Evolution of the output parameters according to the two configrations (Config. 1: left, Config. 2: right)

The results related to Config. 1 show that the region of interest shrinks as σ_2 decreases or h_t increases, and this comes with the decrease of ΔE_z. On the other hand, the region of interest is further enlarged as σ_2 increases or h_t decreases, and we notice a clear reduction of ΔE_z.

According to the results related to Config. 2, the region of interest tightens up as f or h_t increases, and this comes with a drastic decrease of ΔE_z. Contrarily, the region of interest is further expanded as f or h_t decreases, and we observe a noticeable rise of ΔE_z.

5. Conclusion

In this paper, an extension of the infinitesimal model given in [6] has been obtained for a half-wave dipole. Both infinitesimal and extended models have been compared and have shown similar behavior in far-field if the dipolar moment equivalence is respected.

In addition to this results, we proposed a new way to compute critical distances which was used in the case of the extended model. Consequently, Section 4 highlights the scenarios in which the two-ray model could be sufficient for the link estimation. By the means of the maps shown in Fig. 5, it is possible for a radio-link designer to decide whether to use the model in (1) or the one in (2), depending on the range of $\Delta\rho$ and ΔE_z. Should the region of interest and the gap between the two models are large enough, the use of a complete model to predict the surface component would become necessary.

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