Effective model of propagation across a metasurface with resonant Mie inclusions

Kim Pham[1] and Agnès Maurel [2]

[1] IMSIA, CNRS, ENSTA Paris, 828 Bd des Maréchaux, 91732 Palaiseau, France
[2] Institut Langevin, CNRS, ESPCI, 1 rue Jussieu, 75005 Paris, France,
*corresponding author, E-mail: kim.pham@ensta-paris.fr

Abstract

We study the scattering of waves by a single row of resonant inclusions, of the Mie type. An effective model based on matched asymptotic analyzed is used to account for the small thickness of the array. Hence, instead of the effective bulk parameters (permittivity and permeabilty), we end up with interface parameters entering in jump conditions for the electromagnetic fields; among these parameters, one is frequency dependent and encapsulates the resonant behavior of the inclusions. Our effective model is validated by comparison with results of full wave calculations.

1. Introduction

Starting in the 1990's with the pioneering work of [1] in the context of elasticity, resonant structures with subwavelength unit cells have been proposed in the context of electromagnetism [2], and in a unified mathematical context [3]. In this case, the resonances are attributable to an inclusion placed in the unit cell and presenting a high contrast in its material properties with respect to the surrounding matrix. These resonances often referred to as Mie resonances occur at frequencies producing a wavelength in the inclusion comparable to the inclusion size (and this size is much smaller than the incident wavelength). The ability of these so-called locally resonant structures to forbid the wave propagation has been exhibited and the forbidden band gaps have been interpreted in terms of an effective negative parameter being the mass density in elasticity and the permeability in electromagnetism. Since then, locally resonant materials have been intensively studied for applications including the design of efficient wave shields, see *e.g.* [3] or absorbers of small thicknesses, see *e.g.* [4].

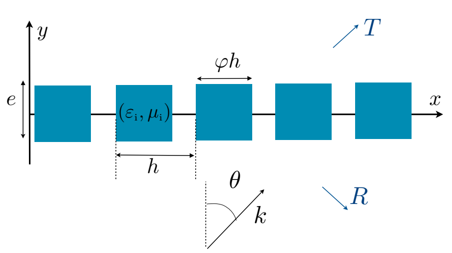

Figure 1: Scattering by an array of locally resonant inclusions with permittivity ε_i and permeability μ_i (in the air).

Motivated by the design of compact metamaterial devices and following the intuitive argument that each local resonator vibrates almost like an independent unit, structures involving a single row of resonators or few rows have been thought, see *e.g.* [5]. In this case, interrogating the bulk response of the device becomes questionable. Indeed, when the number of cells is too small, the metamaterial device is dominated by boundary layer effects and the response of its bulk, in terms of effective parameters, is not pertinent anymore; the failure of the effective medium theories for thick structures has been illustrated in [6]. Here, we extend these works to the case of an array composed of a single row of locally resonant inclusions (Fig. 1).

2. The actual and the effective problems

We consider transverse electric polarized waves propagating in the air and interacting with a row of inclusions with relative permittivity ε_i and permeability μ_i (and we set for the air $\varepsilon = \mu = 1$). In the case where $\varepsilon_i\mu_i \gg 1$, the wavelength inside the inclusions is much smaller than in the air, and resonances of the Mie type are possible. In this case, asymptotic analysis can be conducted which result in an effective interface model, see [7], where the array of resonant inclusions are replaced by jump conditions of the form

$$\left\{ \begin{array}{l} [\![E]\!]_e = h\mathcal{B}\, \overline{\partial y E}, \\ [\![\partial_y E]\!]_e = h\mathcal{S}\, \overline{\partial_{yy} E} + h\mathcal{C}\, \partial_{xx}\overline{E} - h\mathcal{D}(k)\, k^2\, \overline{E}, \end{array} \right. \tag{1}$$

where we defined, for any field f, $[\![f]\!]_e \equiv f^+ - f^-$ and $\overline{f} \equiv (f^+ + f^-)/2$, with $f^\pm \equiv f(x, \pm e/2)$. The effective parameters $(\mathcal{B}, \mathcal{C}, \mathcal{S})$ depend only on the geometry of the inclusions while $\mathcal{D}(k)$ has an additional dependence on the frequency and it is the parameter which encapsulates the possible resonances of the Mie type. For instance, for rectangular shape of inclusion, as we shall consider in the numerical example, we have $\mathcal{S} = \frac{e}{h}(1-\varphi)$, $\mathcal{B} = \frac{e/h}{1-\varphi} - \frac{2}{\pi}\log\left(\cos\frac{\pi\varphi}{2}\right)$, $\mathcal{C} \simeq \frac{\pi}{8}(1-\varphi)^2\left(1 - e^{-\frac{8e}{(1-\varphi)\pi h}}\right)$ and

$$\mathcal{D}(k) = \varepsilon_i \frac{e\varphi}{h}\left[1 - \sum_n \alpha_n^2 \frac{k_i^2}{k_i^2 - k_{i,n}^2}\right], \tag{2}$$

with for $n = (n_1, n_2)$, $k_{i,n}^2 = \left(\frac{n_1\pi}{e}\right)^2 + \left(\frac{n_2\pi}{h\varphi}\right)^2$ the wavenumbers at the resonances and $\alpha_n = \frac{8}{\pi^2 n_1 n_2}$.

3. Fano resonances

In our homogenized problem, the region $|y| < e/2$ is disregarded and the wavefield satisfies (1) for $|y| > e/2$; a solution of this problem for an incident plane wave at oblique incidence θ reads

$$E = \begin{cases} \left[e^{\mathrm{i}k(y+e/2)\cos\theta} + Re^{-\mathrm{i}k(y+e/2)\cos\theta}\right] \; e^{\mathrm{i}kx\sin\theta}, \\ Te^{\mathrm{i}k(y-e/2)\cos\theta} \; e^{\mathrm{i}kx\sin\theta}, \end{cases} \tag{3}$$

for $y < -1/2$ and $y > e/2$ respectively. Next, applying the jump conditions (1) yields the scattering coefficient

$$\begin{cases} R = -\dfrac{1}{2}\left(\dfrac{z_1}{z_1^*} - \dfrac{z_2}{z_2^*}\right), \quad T = \dfrac{1}{2}\left(\dfrac{z_1}{z_1^*} + \dfrac{z_2}{z_2^*}\right), \\ z_1 \equiv 1 + \dfrac{ikh}{2}\,\mathcal{B}\cos\theta, \\ z_2 \equiv \cos\theta + \dfrac{ikh}{2}\left(\mathcal{S}\cos^2\theta + \mathcal{C}\sin^2\theta + \mathcal{D}(k)\right), \end{cases} \tag{4}$$

where z^* denotes the complex conjugate of z.

We report in Fig. 2 the variations of the reflection coefficients against the dimensionless frequency kh, R^{num} being computed numerically and R from (4) (the incidence is $\theta = 40°$). In the range $kh \in [0, 1]$, the first monopolar resonance for $n = (1, 1)$ in (2) is visible (it occurs for $k_i h = 8.9$ whence $kh \simeq 0.9$, resulting in a wavelength within the inclusions being about half the inclusion length). The agreement between R^{num} and its homogenized counterpart R is good, about 2% and this accuracy is the same for any incidence θ up the grazing incidence (results not reported). The resonance of the inclusions produces sharp variations of the reflection with a typical Fano resonance curve characterized by a perfect transmission followed by a perfect reflection. Results including the attenuation due to losses in the dielectric will be presented, with the same accuracy of the effective model to reproduce the scattering properties in the actual problem.

4. Concluding remarks

There are at least two natural extensions of the present study that we shall discuss. We have considered waves in a binary structures but the present work can be extended to ternary structure, as proposed in [8]. This extension may be of particular interest when locally resonant materials are considered in the acoustic case; as previously said, this is obtained in practice considering binary or ternary structures mixing fluids and elastic materials, thus for which the conversions between shear and longitudinal waves have to be accounted for. We believe that this is the key point to properly describe the negative index material reported in this context [9].

The second extension is technically much demanding. Our result has a serious limitation, already mentioned in [2]. The analysis holds for resonances with non zero mean value (namely, n_1 and n_2 odd in (2)). Although these resonances have a higher quality factor and thus are much more sensitive to losses, the accuracy of the homogenized solution would be enhanced if one is able to account for them. This requires to develop the model up to higher order and this is not incremental.

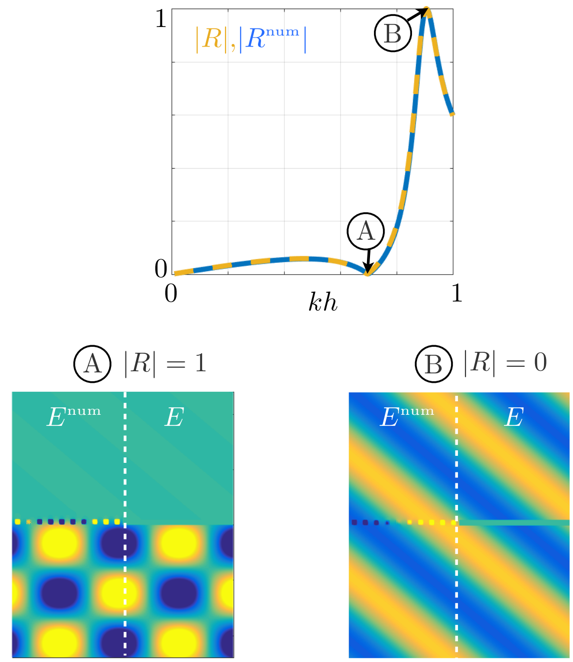

Figure 2: Fano resonance of a row of resonant Mie scatterers in the air ($\varepsilon = 100$, $\varphi = 0.5$, $e/h = 0.5$). Reflection coefficient $|R|$ against kh computed numerically (plain line) and from (4) (dotted line). The wavefields A at $kh = 0.7$ realizing perfect reflection and B at $kh = 0.9$ realizing perfect transmission (numerics and from (3) in both cases).

References

[1] J.-L. Auriault, Curr. Topics Ac. Res. I, 63-90, 1994.

[2] D. Felbacq and G. Bouchitté, PRL **94**(18),183902, 2005.

[3] V.V. Zhikov, Sbornik: Math., **191**(7), 973, 2000.

[3] C. Goffaux *et al.*, PRL, **88**(22), 225502, 2002.

[4] T. Brunet, J. Leng, and O. Mondain-Monval, **342**(6156), 323–324, 2013.

[5] H. Zhao, J. Wen, D. Yu and X. Wen, J. Appl. Phys., **107**(2), 023519, 2010.

[6] M. Lapine, R.C. McPhedran and C.G. Poulton, Phys. Rev. B, **93**(23), 235156, 2016.

[7] K. Pham, A. Maurel and J.-J. Marigo, JMPS, **106**, 80-94, 2017.

[8] Z. Liu, X. Zhang, Y. Mao, Y.Y. Zhu, Z. Yang, C.T. Chan, and P. Sheng, Science, **289**(5485, 1734-1736, 2000.

[9] Y. Ding, Z. Liu, C. Qiu, and J. Shi, Jing, Phys. Rev. Lett., **99**(9) 093904, 2007.