Scattering by a dielectric cylinder with arbitrary cross section using Pseudo-spectral Modal Method

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Abstract

We present a new semi-analytical formulation for diffraction by structured cylinders. A pseudo-spectral modal method is used to solve the Maxwell equations written in curvilinear coordinates. The program is compared with the numerical results obtained with finite element method using Comsol Multiphysics.

1. Introduction

The last two decades have been marked by the study and fabrication of structured materials to control light. To solve complex electromagnetic problems, numerical methods are widely used. We propose a semi-analytical model based on the pseudo-spectral method using the Chebyshev differentiation matrix [1, 2, 3]. In this method, Maxwell's equations are written in their covariant form and a change of coordinates is used to transform an arbitrary cross section cylinder into a circular cross section one. Since the boundary of the cylinder coincide with a coordinate surface, writing the boundary conditions is very easy.

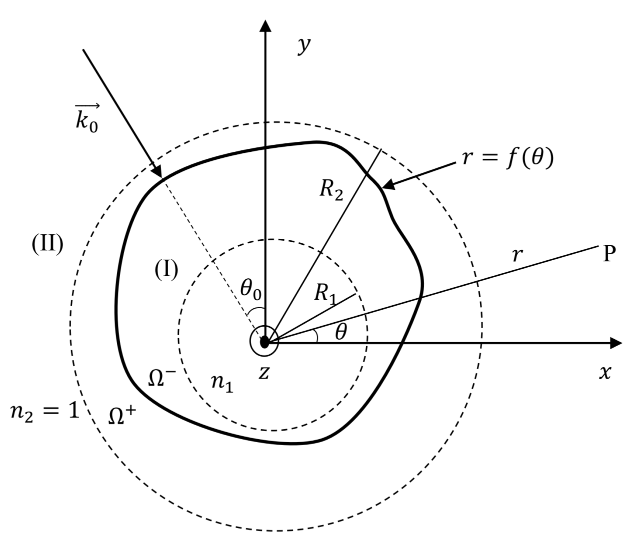

Figure 1: Geometry of the problem.

Since the structure has cylindrical symmetry and 2π periodic with θ, in the usual cylindrical coordinate the incident field is given by :

$$E_z^i(r, \theta) = E_0 \exp[ik_0 r \sin(\theta - \theta_0)] \tag{1}$$

2. resolution

2.1. New Coordinates

The change of coordinates transforms a section with arbitrary contour into a circular section. In the new coordinates, the space is divided into four regions separated by circles with radii $u_1 = R_1$ and $u_3 = R_2$. The circle with radius u_2 coincides with the profile under study. We have :

$$a(u, \theta) = \begin{cases} R_1 + [f(\theta) - R_1]\dfrac{u - u_1}{u_2 - u_1} & \text{for} \quad u_1 \leq u \leq u_2 \\ f(\theta) + [R_2 - f(\theta)]\dfrac{u - u_2}{u_3 - u_2} & \text{for} \quad u_2 \leq u \leq u_3 \end{cases} \tag{2}$$

In order to respect the non singularity of the field at $r = 0$ and the radiation condition for $r \to +\infty$, the field inside the region $r < R_1$ and outside the region $r > R_2$ are respectively developed in Fourier-Bessel and Hankel series.

$$E_z^{(1)}(r, \theta) = \sum_{m=-M}^{M} A_{1m} J_m(k_0 n_1 r) \exp(im\theta) \tag{3}$$

$$E_z^{(3)}(r, \theta) = \sum_{m=-M}^{M} \Big\{ \big[(-i)^m J_m(k_0 r) + A_{3m} H_m^{(2)}(k_0 r)\big] \exp(im\theta) \Big\} \tag{4}$$

2.2. Maxwell's Covariant Equations

In a source free homogeneous medium, considering the angular frequency ω in the harmonic regime $\exp(-i\omega t)$ and with Einstein's convention, the Maxwell-Minkowski equations are written.

$$\begin{cases} \xi^{ijk}\partial_j E_k = +i\omega B^i \\ \xi^{ijk}\partial_j H_k = -i\omega D^i \end{cases} \tag{5}$$

where $i, j, k = 1, 2, 3$. ξ^{ijk} is the Levi-Civita indicator.

The constitutive equations are given by :

$$\begin{cases} B^i = \mu^{ij} H_j \\ D^i = \varepsilon^{ij} E_j \end{cases} \tag{6}$$

where the μ^{ij} and the ε^{ij} depend on the metric associated with new coordinates and on the medium.

2.3. Pseudo spectral method

In regions $\Omega^- = [u_1, u_2]$, $\Omega^+ = [u_2, u_3]$, the unknown function $E_z(u)$ is approximated by the product of a set of suitable Lagrange-interpolating functions C_n and unknown grid point values $E_z(u_n)$ at collocation points u_n as follows [4]:

$$E_z(u) = \sum_{n=0}^{N} C_n(u) E_z(u_n) \qquad (7)$$

Considering Tchebycheff polynomials T_n and Gauss- Lobatto points u_n as basis functions and collocation points, respectively, the explicit form of C_n is:

$$C_n(u) = \frac{(-1)^{n+1}(1-u^2)T'_N(u)}{\sigma_n N^2 (u-u_n)}, \quad u \neq u_n \qquad (8)$$

with $\sigma_0 = \sigma_N = 2$ and $\sigma_n = 1$ for $1 \leq n \leq N-1$. The above expansion is matched with (3) at $u = u_2$ and with (4) at $u = u_3$ which allows to compute the A_{3m} which correspond to the expansion coefficients of the diffacted field. Lastly, we may compute the radar cross defined as.

$$\sigma = \lim_{r\to\infty} 2\pi r \frac{|E^{diff}|^2}{|E^{inc}|^2} \qquad (9)$$

3. Results

For z-invariant cylinders equation (5) reduces to two independent scalar problems with E_z or H_z as unknowns according to the incident plane wave polarization. Numerically, we obtain a matrix equation which is solved for the A_{3m}. For illustrative purpose, we consider the profile defined as $r = \lambda[1 + 0.5\sin(10\theta)]$. The refractive index is $n_1 = 1.5$.

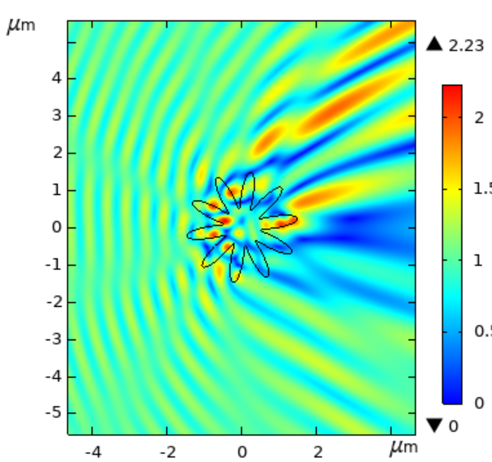

Figure 2: Spatial distribution of the E_z ($\lambda = 1\mu m$).

4. Conclusions

We have proposed a pseudo-spectral modal method for electromagnetic diffraction. We could solve the problem of diffraction by cylinder with arbitrary section which is very difficult with other method. The specificity of this method lies in the fact that the field at infinity can be handle without any Perfect Matched Layer.

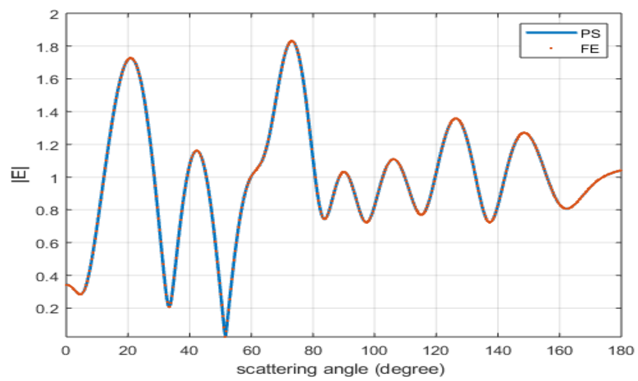

Figure 3: Comparison of the total field surrounding the cylinder at a distance of $r = 2.5\lambda$.

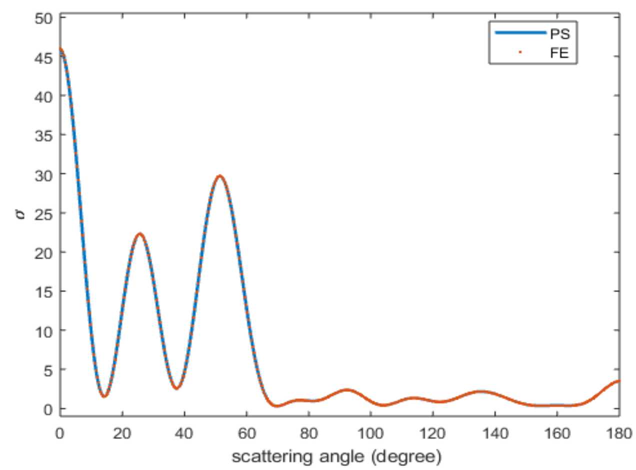

Figure 4: Comparison of the Radar Cross Section obtained with our method and with Comsol.

References

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