Emergent $\mathcal{PT}$ symmetry and quantum fluctuations in a double-quantum-dot circuit QED set-up

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Abstract

Open classical and quantum systems with effective parity-time ($\mathcal{PT}$) symmetry have shown tremendous promise for advances in lasers, sensing, and non-reciprocal devices. However, the microscopic origin of such effective, non-Hermitian models is not well under-stood. In this work, by microscopically modelling a double-quantum-dot-circuit-QED set-up that is realizable in state-of-the-art experiments, we show that a non-Hermitian Hamiltonian emerges naturally, which can be controllably tuned to observe both $\mathcal{PT}$-transition, as well as the effect of quantum fluctuations.

1. The set-up

The schematic diagram of the set-up is given in Fig. 1. The main parts of the set-up consist of the DQD and the two cavities. These are described by the following Hamiltonians

$$\hat{\mathcal{H}}_{DQD} = \frac{\varepsilon}{2}(\hat{c}_1^\dagger \hat{c}_1 - \hat{c}_2^\dagger \hat{c}_2) + t_c(\hat{c}_1^\dagger \hat{c}_2 + \hat{c}_2^\dagger \hat{c}_1) + V \hat{c}_1^\dagger \hat{c}_1 \hat{c}_2^\dagger \hat{c}_2,$$
$$\hat{\mathcal{H}}_C = \omega_0(\hat{b}_1^\dagger \hat{b}_1 + \hat{b}_2^\dagger \hat{b}_2) + \lambda(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1),$$
$$\hat{\mathcal{H}}_{DQD-C} = g_0 \Theta(t)(\hat{c}_1^\dagger \hat{c}_1 - \hat{c}_2^\dagger \hat{c}_2)(\hat{b}_1^\dagger + \hat{b}_1), \quad (1)$$

where $\Theta(t)$ is the Heaviside function. Here, $\hat{c}_{1,2}^\dagger$ denote fermionic creation operators for sites 1 and 2 that model the DQD, ε is the energy difference between the two sites, t_c is the hopping amplitude between the two sites, and $V > 0$ denotes the capacitive charging energy between the two sites. $\hat{b}_{1,2}$ represent the bosonic creation operators for the two cavities, each with frequency ω_0, that are coupled via a number-conserving hopping process with amplitude λ. Closely following the experimental set-ups, the DQD is dipole-coupled to the (first) cavity it is in with strength g_0 when the cavity is switched on at time $t = 0$. Thus, Eq.(1) captures the microscopic model of the DQD and the two cavities.

Each of the three main components is connected to multiple baths. All baths are modelled by Hamiltonians quadratic in bosonic or fermionic creation and annihilation operators. Each cavity is coupled to its own bosonic bath at inverse temperature β, with $\beta\omega_0 \gg 1$. The presence of these baths lead to a cavity decay rate of κ_1 (κ_1) for the first (second) cavity. Each site of the DQD is coupled to its own fermionic lead. The lead coupled to the first (second) site has inverse temperature β and chemical potential μ_1 (μ_2). The strength of coupling to both leads is taken to be same and denoted by Γ. The entire set-up is realizable in state-of-the-art experiments with DQD coupled to circuit-QED cavities. However, to model experimental set-up consistently, we also have to consider that the DQD is dipole-coupled to a phononic bath in the substrate on which it is located [1].

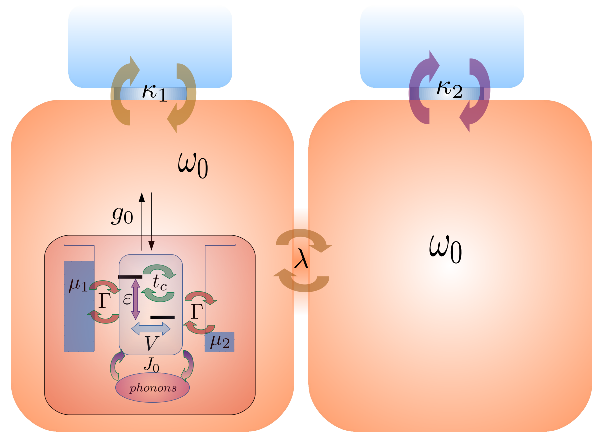

Figure 1: The set-up consists of two cavities connected to each other, with the left cavity having a double-quantum-dot out-of-equilibrium. The double-quantum-dot is configured to give a gain to the left cavity. If the gain is same as the loss of the right cavity, the bosonic system of the cavities have an effective $\mathcal{PT}$ symmetric dynamics.

We look at the parameter regime that ensures a resonant DQD, i.e $\omega_0 = \omega_q$; a weak cavity-DQD couplng, i.e. $g_0\sqrt{n_{\text{photons}}} \lesssim \Gamma$ (n_{photons} is the average number of photons in the cavity coupled to the DQD), and

$$\omega_q/2 \ll -\mu_2, \mu_1 \ll V,$$
$$\kappa_1, \kappa_2, \lambda \ll \Gamma \ll \omega_0. \quad (2)$$

In this parameter regime, the it can be shown that, for $\varepsilon > 0$, the DQD will be population inverted in the steady state in the absence of the cavities. The DQD thus acts as a controllable gain medium [2].

2. The effective $\mathcal{PT}$ symmetry

After integrating out the fermionic part and the bosonic baths, we have the effective equations of motion for the bosonic system,

$$i\frac{d}{dt}\begin{pmatrix}\hat{b}_1\\ \hat{b}_2\end{pmatrix} \simeq \mathbf{H}_{\text{eff}}\begin{pmatrix}\hat{b}_1\\ \hat{b}_2\end{pmatrix} + \begin{pmatrix}\hat{\xi}_A\\ 0\end{pmatrix}. \qquad (3)$$

The 2×2 non-Hermitian Hamiltonian is given by

$$\mathbf{H}_{\text{eff}} = \begin{bmatrix}\omega_0 - i\kappa_1 + i\Gamma\delta & \lambda\,(1-\delta)\\ \lambda & \omega_0 - i\kappa_2\end{bmatrix}, \qquad (4)$$

where $\delta = 2g^2\Delta N_{ss}/\Gamma^2 \ll 1$ by the choice of our parameters and $\Delta N_{ss} = \langle\hat{N}_1\rangle_{ss} - \langle\hat{N}_2\rangle_{ss}$, is the steady-state population inversion in the DQD in the absence of cavities. Here $\langle\hat{N}_1\rangle_{ss}$ ($\langle\hat{N}_2\rangle_{ss}$) is the population of the higher (lower) energy state of the DQD in steady state in absence of the cavity. The time-dependent quantum noise operator $\hat{\xi}_A(t)$, resulting from the DQD gain, has almost zero mean value, $\langle\hat{\xi}_A(t)\rangle \approx 0$, and a Lorenzian power spectrum for the variance, i.e.

$$\langle\hat{\xi}_A^\dagger(t)\hat{\xi}_A(t')\rangle \simeq g^2\langle\hat{N}_1\rangle_{ss}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{2\Gamma e^{i\omega(t-t')}}{(\omega-\omega_0)^2+\Gamma^2}. \quad (5)$$

In all of the equations above, the expectation value represents quantum statistical average, i.e. $\langle O\rangle = \text{Tr}(\rho_{\text{tot}}O)$ where ρ_{tot} is the density matrix for the initial state of the whole set-up. The effective Hamiltonian in Eq. 4 can be tuned to observe both active and passive $\mathcal{PT}$ transition just by tuning the parameters ε and t_c of the DQD and by tuning the parameter λ of the cavities. As can be easily seen from Eq. 3, the effect of the various phases will be observed in the dynamics of the complex quadratures $\langle\hat{b}_\ell(t)\rangle$ ($\ell = 1, 2$). However, our complete microscopic derivation has lead to occurrence of the additional noise term $\hat{\xi}_A(t)$, in consistency with fluctuation-dissipation theorem. The presence of $\hat{\xi}_A(t)$ lets us study the effect of quantum fluctuations on the dynamics of $\mathcal{PT}$-symmetric and $\mathcal{PT}$-broken phases.

As an example, here, in Fig. 2, we show the effect of quantum fluctuations on the active $\mathcal{PT}$-symmetric phase where the eigenvalues of $\mathbf{H}_{\text{eff}}$ are real. The dynamics is shown starting from a state where the cavity with the DQD (gain cavity) is in a coherent state with the other cavity (loss cavity) is empty. The amplitudes of the complex quadratures show periodic oscillations characteristic of the $\mathcal{PT}$-symmetric phase. However, the quantum fluctuations of the quadratures, given by $\langle\hat{n}_\ell(t)\rangle - |\langle\hat{b}_\ell(t)\rangle|^2$, where $\hat{n}_\ell(t) = \hat{b}_\ell^\dagger(t)\hat{b}_\ell(t)$, show linear growth superimposed with oscillations. Similar behavior is seen in the photon current $I(t) = \lambda\text{Im}(\langle\hat{b}_2^\dagger(t)\hat{b}_1(t)\rangle)$, where Im() refers to imaginary part.

3. Conclusions and further work

The set-up of two coupled cavities with a voltage biased DQD in one of them provides a perfect test bed for exploring effective $\mathcal{PT}$-symmetry in the quantum regime, and

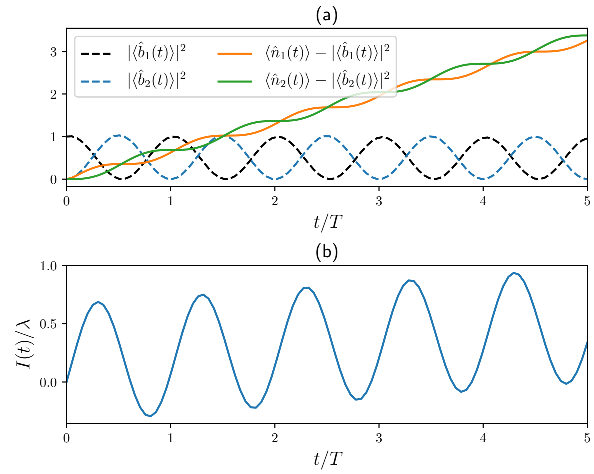

Figure 2: Oscillatory dynamics of the cavity operator expectation values in the $\mathcal{PT}$-symmetric regime with empty loss cavity and a coherent state in the gain cavity. (a) The quadrature magnitudes show oscillations with period $T_\Lambda = 2\pi/\Delta\Lambda$. On the other hand, quantum fluctuations show a linear growth with superimposed oscillations with the same period. (b) Dynamics of the photonic current $I(t)$ also shows linear growth with superimposed oscillations.

explore the effect of quantum fluctuations. While not discussed in the above summary, we have also explored the effect of passive $\mathcal{PT}$-transition in input-output experiments. Further, we have been able to show that as a consequence of the passive $\mathcal{PT}$-transition, loss induced lasing can be observed in the set-up.

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