Full-vectorial polynomial modal method for circular waveguides. Application to reflection and diffraction at the end of radially inhomogeous cylinders.

Gérard Granet

Université Clermont Auvergne,
CNRS,
SIGMA Clermont,
Institut Pascal,
F-63000 CLERMONT-FERRAND, FRANCE
E-mail: gerard.granet@uca.fr

Abstract

We develop a modal method for radially inhomenous waveguide. The formulation is derived with the two transverse components of the magnetic field. The algebraic eigensystem is solved by using Tchebycheff polynomials and a tau method.

1. Introduction

Cylindrical objects are the building blocks of photonic crystals, metamaterials, and fibers. Everyday components like waveguides, or resonators are created on their basis. More recently, plasmonic nano wires have also been studied [1]. Another promising area of research in nanophotonics is the tight focus that is formed using dielectric cylinders with diameters about wavelength [2], [3]. Nowadays, many efficient commercial codes are able to simulate efficiently such structures. However researchers often feel the need to develop in house codes in order to be able to completely master all the numerical parameters of a given simulation. Another reason to develop original codes is that one can get faster tools since they can be designed for a specific need. We are interested in the analysis of radially piece wise inhomogeneous circular waveguides using a polynomial modal method which is known to be robust with regards to materials properties because it enforces in an exact manner the interface boundary conditions. The originality of the method lies in the theoretical formulation of the spectral problem and its solution using a Galerkin method with Tchebychelf polynomials as expansion and test functions [4],[5],[6]. Once the eigen-solutions are computed, the rest of the solving follows the usual steps of any modal method. Fig 1 shows a typical structure that can be analysed with our code.

2. Theory

2.1. Eigenvalue equation

A quarter of the cross section of one step index circular wave-guide is shown in Fig.(2). In case a radial radiation condition is needed, PML may be introduced through complex coordinates. Let us consider a time-harmonic magnetic

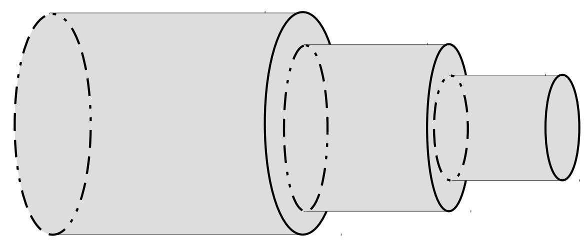

Figure 1: sketch of a multi sectional waveguide.

field such that:

$$\mathbf{H}(r,\phi,z) = \begin{bmatrix} H_r(r)\cos(m\phi) \\ H_\phi(r)\sin(m\phi) \\ H_z(r,\phi) \end{bmatrix} \exp j(\omega t - \gamma z) \tag{1}$$

In an homogeneous medium with relative permittivity ϵ_r and in the cylindrical coordinate system the vector wave equation for the magnetic field reduces to three scalar partial differential equations of the form

$$\begin{aligned} &\Delta H_r + \left(-\frac{H_r}{r^2} - \frac{2}{r^2}\frac{\partial H_\phi}{\partial_\phi}\right) = -k_0^2\epsilon_r \\ &\Delta H_\phi + \left(-\frac{H_\phi}{r^2} + \frac{2}{r^2}\frac{\partial H_r}{\partial_\phi}\right) = -k_0^2\epsilon_r \\ &\Delta H_z = -k_0^2\epsilon_r \end{aligned} \tag{2}$$

where k_0 is the wave number and where Δ represents the Laplacian of a scalar that in cylindrical coordinates takes the form of:

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2} \tag{3}$$

Taking into account the special form of the field given by Eq(1), we get:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + k_0^2\epsilon_r\frac{K}{r^2}\right)\begin{bmatrix} H_r \\ H_\phi \end{bmatrix} = \gamma^2\begin{bmatrix} H_r \\ H_\phi \end{bmatrix} \tag{4}$$

$$K = -\begin{bmatrix} m^2+1 & 2m \\ 2m & m^2+1 \end{bmatrix} \tag{5}$$

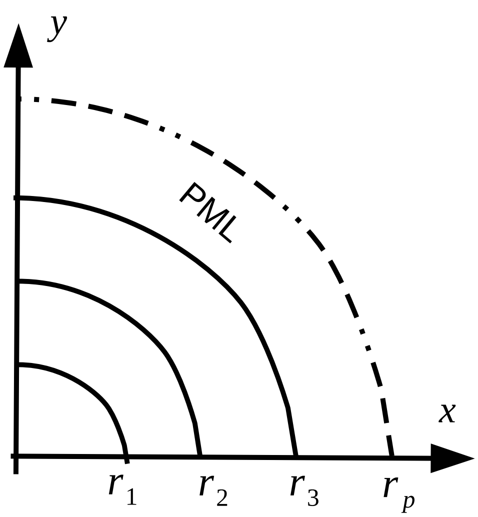

Figure 2: A quarter of the cross section of a radially piecewise inhomogeneous waveguide. The PML is backed by a PEC.

2.2. Boundary conditions

The magnetic field vector needs also to satisfy boundary conditions at coordinate surfaces $r = r_i$, at origin $r = 0$ and at $r = r_p$ which is the outer radius of the PML region.
At $r = r_i$

- H_r and H_ϕ are continuous
- $\dfrac{\partial H_r}{\partial r}$ is conthiinuous
- $\dfrac{1}{\epsilon_r}\left(\dfrac{\partial (rH_\phi)}{\partial r} + mH_r\right)$ is continuous

At $r = r_0$

- $H_r = H_\phi = 0$ if $m \neq 1$
- $\dfrac{\partial H_r}{\partial r} = \dfrac{\partial H_\phi}{\partial r} = 0$ if $m = 1$

At $r = r_p$

- $\dfrac{\partial (rH_\phi)}{\partial r} = 0$

2.3. Numerical solution

In a first step, in each homogeneous region, H_r and H_ϕ are approximated by a linear combination of Tchebycheff polynomials T_q:

$$\begin{array}{l} H_r(r) = \sum_{p=0}^{M-1} H_{rp}T_q(r) + \sum_{p=M}^{M+1} H_{rp}T_p(r) \\ H_\phi(r) = \sum_{q=0}^{M-1} H_{\phi q}T_q(r) + \sum_{q=M}^{M+1} H_{\phi q}T_q(r) \end{array} \quad (6)$$

Then Eq(4) is projected onto $2M$ Tchebycheff polynomials. Lastly an eigen-system, valid in the whole computational domain, is obtained by adding the relations which express the boundary conditions.

3. Conclusions

Our talk will be devoted to the detailed presentation of the polynomial modal method as applied to cylindrical structures. We will illustrate the effectiveness of the method through many examples. More specifically we will investigate the near field scattered at the end of a multi-sectional dielectric wave-guide

References

[1] T.Häyren, J.Rosenkrantz de Lasson, N.Gregersen, Open-geometry Fourier modal method:modeling nanophotonic structures in infinite domains, J. Opt. Soc. Am. A 33:1298–1306,2016.

[2] J.Zelgowski, A.Abdurrochman, F. Mermet, P. Pfeiffer, J. Fontaine, S. Lecler, Photonic jet subwavelength etching using a shaped optical fiber tip, Opt.Lett. 41:2073–2076,2016

[3] Y. Zhang 1, S. Li, Y. Zhu, Y. Zhuang, T. Suyama, C. Zheng ,Y. Okuno, Periodic transmission of circular binary Fresnel zone plates with etching depth and substrate, PIER Lett.,40:93–105,2013.

[4] R.Song, J.Zhu, X.Zhang, Full-vectorial modal analysis for circular optical wavegudes based on the multidomain Chebyshev pseudospectral method. J. Opt. Soc. Am. B 27:1722–1730,2010.

[5] K.Edee, M.Aboud, G.Granet, J.F. Cornet, J.Dauchet, Reformulation of the modal method based on Gegenbauer polynomial expansion for cylindrical structures with arbitrary cross sections, J. Opt. Soc. Am. A 31:2174–2178,2014.

[6] K.Edee, M.Aboud, G.Granet, J.F. Cornet, N.A. Gippius, Mode solver based on Gegenbauer polynomial expansion for cylindrical structures with arbitrary cross sections J. Opt. Soc. Am. A 31: 667–676,2014.